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On Influence and Contractions in Defeasible Logic Programming

Diego R. García[1], Sebastián Gottifredi[1], Patrick Krümpelmann[2], Matthias Thimm[2], Gabriele Kern-Isberner[2], Marcelo A. Falappa[1], and Alejandro J. García[1]

[1] Universidad Nacional del Sur, Bahía Blanca, Argentina
[2] Technische Universität Dortmund, Germany

Abstract. In this paper, we investigate the problem of contraction in Defeasible Logic Programming (DeLP), a logic-based approach for defeasible argumentation. We develop different notions of contraction based on both, the different forms of entailment implicitly existent in argumentation-based formalisms and the influence literals exhibit in the reasoning process. We give translations of widely accepted rationality postulates for belief contraction to our framework. Moreover we discuss on the applicability of contraction for defeasible argumentation and the role of influence in this matter.

1 Introduction

While most of the past work in argumentation has been done on the study of representation and inferential properties of different frameworks, the problem of belief change—one of the most important problems in knowledge representation—has not been investigated in depth so far, see [3] for a survey. Revision and the dynamics of beliefs in general have been studied for classical logics since the seminal paper [1]. There are also some proposals for dealing with belief change in non-classical logics, e. g. for defeasible logic in [2]. Here we consider the problem of contraction in the framework of *Defeasible Logic Programming* (DeLP) [4].

In DeLP, using a dialectical procedure involving arguments and counterarguments, literals can be established to be *warranted*, meaning that there are *considerable* grounds to believe the literal being true. The straightforward approach to define the success of contracting a defeasible logic program $\mathcal{P}$ by a literal l is to demand that l is not warranted anymore. This is similar to the approaches taken for classical theories [1] where success is defined in terms of non-entailment. In DeLP, however, there are three basic notions of "entailment": derivation (there are rules in $\mathcal{P}$ that allow to derive l), argument (there is an argument for l), and warrant (there is an undefeated argument for l). These notions lead to different alternatives of defining the success of a contraction. In addition to these notions of entailment, we also investigate the notion of *influence* in order to gain more insights into the problem of contraction. Due to the dialectical nature of the reasoning process employed in DeLP a literal can exhibit influence on the warrant status of other literals independently of its own warrant status. Contracting a program $\mathcal{P}$ by a literal l using the notion of warrant to define success still allows the dialectical procedure to use arguments for l in changing the warrant status of another literal.

This paper is organized as follows. In Section 2 we give a brief overview on the basic notions of Defeasible Logic Programming and continue with a thorough investigation of the notion of influence in Section 3. We apply the developed notions thereafter for investigating rationality postulates for contraction in our framework in Section 4. In Section 5 we conclude with a discussion of related and further work.

2 Defeasible Logic Programming

A single atom h or a negated atom $\sim h$ is called a literal or *fact*. Rules are divided into strict rules $h \leftarrow B$ and defeasible rules $h \prec B$ with a literal h and a set of literals B. A literal h is *derivable* from a set of facts and rules X, denoted by $X \mid\!\sim h$, iff it is derivable in the classical rule-based sense, treating strict and defeasible rules equally. A set X is *contradictory*, denoted $X \mid\!\sim \perp$, iff both $X \mid\!\sim h$ and $X \mid\!\sim \sim h$ holds for some h. A literal h is *consistently derivable* by X, denoted by $X \mid\!\sim^c h$, iff $X \mid\!\sim h$ and $X \not\mid\!\sim \perp$. A *defeasible logic program* (de.l.p.) $\mathcal{P}$ is a tuple $\mathcal{P} = (\Pi, \Delta)$ with a non-contradictory set of strict rules and facts Π and a set of defeasible rules Δ. We write $\mathcal{P} \mid\!\sim h$ as a shortcut for $\Pi \cup \Delta \mid\!\sim h$.

Definition 1 (Argument, Subargument). *Let h be a literal and let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. Then $\langle \mathcal{A}, h \rangle$ with $\mathcal{A} \subseteq \Delta$ is an* argument *for h iff $\Pi \cup \mathcal{A} \mid\!\sim^c h$ and $\mathcal{A}$ is minimal wrt. set inclusion. A $\langle \mathcal{B}, q \rangle$ is a* subargument *of $\langle \mathcal{A}, h \rangle$ iff $\mathcal{B} \subseteq \mathcal{A}$.*

$\langle \mathcal{A}_1, h_1 \rangle$ is a *counterargument* to $\langle \mathcal{A}_2, h_2 \rangle$ at literal h, iff there is a subargument $\langle \mathcal{A}, h \rangle$ of $\langle \mathcal{A}_2, h_2 \rangle$ such that $\Pi \cup \{h, h_1\}$ is contradictory.

In order to deal with counterarguments, a formal comparison criterion among arguments is used. Our results are independent of its choice, but as en example we use the *generalized specificity* relation $\succ$ [4]. Then, $\langle \mathcal{A}_1, h_1 \rangle$ is a *defeater* of $\langle \mathcal{A}_2, h_2 \rangle$, iff there is a subargument $\langle \mathcal{A}, h \rangle$ of $\langle \mathcal{A}_2, h_2 \rangle$ such that $\langle \mathcal{A}_1, h_1 \rangle$ is a counterargument of $\langle \mathcal{A}_2, h_2 \rangle$ at literal h and either $\langle \mathcal{A}_1, h_1 \rangle \succ \langle \mathcal{A}, h \rangle$ (*proper defeat*) or $\langle \mathcal{A}_1, h_1 \rangle \not\succ \langle \mathcal{A}, h \rangle$ and $\langle \mathcal{A}, h \rangle \not\succ \langle \mathcal{A}_1, h_1 \rangle$ (*blocking defeat*).

Definition 2 (Acceptable Argumentation Line). *Let $\Lambda = [\langle \mathcal{A}_1, h_1 \rangle, \ldots, \langle \mathcal{A}_m, h_m \rangle]$ be a finite sequence of arguments. Λ is called an* acceptable argumentation line*, iff 1.) every $\langle \mathcal{A}_i, h_i \rangle$ with $i > 1$ is a defeater of $\langle \mathcal{A}_{i-1}, h_{i-1} \rangle$ and if $\langle \mathcal{A}_i, h_i \rangle$ is a blocking defeater of $\langle \mathcal{A}_{i-1}, h_{i-1} \rangle$ and $\langle \mathcal{A}_{i+1}, h_{i+1} \rangle$ exists, then $\langle \mathcal{A}_{i+1}, h_{i+1} \rangle$ is a proper defeater of $\langle \mathcal{A}_i, h_i \rangle$, 2.) $\Pi \cup \mathcal{A}_1 \cup \mathcal{A}_3 \cup \ldots$ is non-contradictory, 3.) $\Pi \cup \mathcal{A}_2 \cup \mathcal{A}_4 \cup \ldots$ is non-contradictory, and 4.) no $\langle \mathcal{A}_k, h_k \rangle$ is a subargument of some $\langle \mathcal{A}_i, h_i \rangle$ with $i < k$.*

In DeLP a literal h is *warranted*, if there is an argument $\langle \mathcal{A}, h \rangle$ which is non-defeated in the end. To decide whether $\langle \mathcal{A}, h \rangle$ is defeated or not, every acceptable argumentation line starting with $\langle \mathcal{A}, h \rangle$ has to be considered.

Definition 3 (Dialectical Tree). *Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and let $\langle \mathcal{A}_0, h_0 \rangle$ be an argument. A* dialectical tree *for $\langle \mathcal{A}_0, h_0 \rangle$, denoted $\mathcal{T}\langle \mathcal{A}_0, h_0 \rangle$, is defined as follows: The root of $\mathcal{T}\langle \mathcal{A}_0, h_0 \rangle$ is $\langle \mathcal{A}_0, h_0 \rangle$. Let $\langle \mathcal{A}_n, h_n \rangle$ be a node in $\mathcal{T}\langle \mathcal{A}_0, h_0 \rangle$ and let $[\langle \mathcal{A}_0, h_0 \rangle, \ldots, \langle \mathcal{A}_n, h_n \rangle]$ be a sequence of nodes. Let $\langle \mathcal{B}_1, q_1 \rangle, \ldots, \langle \mathcal{B}_k, h_k \rangle$ be the defeaters of $\langle \mathcal{A}_n, h_n \rangle$. For every defeater $\langle \mathcal{B}_i, q_i \rangle$ with $1 \leq i \leq k$ such that $[\langle \mathcal{A}_0, h_0 \rangle, \ldots, \langle \mathcal{A}_n, h_n \rangle, \langle \mathcal{B}_i, q_i \rangle]$ is an acceptable argumentation line, the node $\langle \mathcal{A}_n, h_n \rangle$ has a child $\langle \mathcal{B}_i, q_i \rangle$. If there is no such $\langle \mathcal{B}_i, q_i \rangle$, $\langle \mathcal{A}_n, h_n \rangle$ is a leaf.*

In order to decide whether the argument at the root of a given dialectical tree is defeated or not, it is necessary to perform a *bottom-up*-analysis of the tree. Every leaf of the tree is marked "undefeated" and every inner node is marked "defeated", if it has at least one child node marked "undefeated". Otherwise it is marked "undefeated". Let $\mathcal{T}^*\langle \mathcal{A}_0, h_0 \rangle$ denote the marked dialectical tree of $\mathcal{T}\langle \mathcal{A}_0, h_0 \rangle$. We call a literal h *warranted* in a DeLP $\mathcal{P}$, denoted by $\mathcal{P} \mid\!\sim_w h$, iff there is an argument $\langle \mathcal{A}, h \rangle$ for h in $\mathcal{P}$ such that the root of the marked dialectical tree $\mathcal{T}^*\langle \mathcal{A}, h \rangle$ is marked "undefeated". Then $\langle \mathcal{A}, h \rangle$ is a *warrant* for h.

We will need some further notation in the following. Let $\mathcal{P} = (\Pi, \Delta)$ and $\mathcal{P}' = (\Pi', \Delta')$ be some programs and let r be a rule (either defeasible or strict). $\mathcal{P}$ is a subset of $\mathcal{P}'$, denoted by $\mathcal{P} \subseteq \mathcal{P}'$, iff $\Pi \subseteq \Pi'$ and $\Delta \subseteq \Delta'$. It is $r \in \mathcal{P}$ if either $r \in \Pi$ or $r \in \Delta$. We also define $\mathcal{P} \cup r =_{def} (\Pi, \Delta \cup \{r\})$ and $\mathcal{P} \cup \mathcal{A} =_{def} (\Pi, \Delta \cup \mathcal{A})$ for an argument $\mathcal{A}$.

3 Influence in Defeasible Logic Programs

It may be the case that an argument $\mathcal{A}$ for l is defeated in its own dialectical tree and not defeated in another tree [6]. Thus, these undefeated arguments for l may exhibit some *influence* one the marking status of arguments for another literal. Hence, by employing a contraction operation that bases its success only on the warrant status of the literal under consideration, it should be kept in mind that this literal might still have influence on the reasoning behavior in the contracted program. We therefore continue by investigating different notions of influence in more depth. Our first approach bases on an observation made when considering the removal of arguments for the literal l that has to be contracted.

Definition 4 (Trimmed Dialectical Tree). *Let $\mathcal{T}$ be some dialectical tree and l a literal. The l-*trimmed dialectical tree *$\mathcal{T} \setminus_t l$ is the same as $\mathcal{T}$ but every subtree $\mathcal{T}'$ of $\mathcal{T}$ with root $\langle \mathcal{A}_1, h \rangle$ and $\mathcal{A}_2 \subseteq \mathcal{A}_1$ such that $\langle \mathcal{A}_2, l \rangle$ is an argument for l is removed from $\mathcal{T}$.*

Note, that a trimmed dialectical tree is not a dialectical tree (as it is not complete) but that the marking procedure is still applicable in the same way.

Proposition 1. *Let l be a literal and $\mathcal{T}$ a dialectical tree. If $\mathcal{T} \setminus_t l$ is not empty and the marking of the root of $\mathcal{T}^*$ differs from the marking of the root of $(\mathcal{T} \setminus_t l)^*$ then there is an argument $\langle \mathcal{A}, k \rangle$ with $\mathcal{A}' \subseteq \mathcal{A}$ such that $\langle \mathcal{A}', l \rangle$ is an argument l and $\langle \mathcal{A}, k \rangle$ is undefeated in $\mathcal{T}^*$.*

Proposition 1 establishes that only an argument for l that is undefeated in $\mathcal{T}^*$ can possibly exhibit some influence. This leads to our first and most general definition of influence.

Definition 5 (Argument Influence $\mathcal{I}_\mathcal{A}$). *A literal l has* argument influence *in $\mathcal{P}$, denoted by $l \leadsto^\mathcal{A} \mathcal{P}$, if, and only if there is an argument $\langle \mathcal{A}_1, h\rangle$ with $\mathcal{A}_2 \subseteq \mathcal{A}_1$ such that $\langle \mathcal{A}_2, l\rangle$ is an argument for l and $\langle \mathcal{A}_1, h\rangle$ is a node in a dialectical tree $\mathcal{T}^*$ and $\langle \mathcal{A}_1, h\rangle$ is marked "undefeated" in $\mathcal{T}^*$.*

However, it is not the case that every argument that contains a subargument for l and is undefeated in some dialectical tree necessarily exhibits reasonable influence. This leads to our next notion of influence that only takes arguments into account which, on removal, will change the marking of the root.

Definition 6 (Tree Influence $\mathcal{I}_\mathcal{T}$). *A literal l has* tree influence *in $\mathcal{P}$, denoted by $l \leadsto^\mathcal{T} \mathcal{P}$, if and only if there is a dialectical tree $\mathcal{T}^*$ such that either 1.) the root's marking of $\mathcal{T}^*$ differs from the root's marking of $(\mathcal{T} \setminus_t l)^*$ or 2.) the root of $\mathcal{T}^*$ is marked "undefeated" and $(\mathcal{T} \setminus_t l)^*$ is empty.*

In order to establish whether a literal l exhibits *tree influence* every dialectical tree is considered separately. But recall, that for a literal h being warranted only the existence of a single undefeated argument is necessary. These considerations result in our final notion of *warrant influence.*

Definition 7 (Warrant Influence $\mathcal{I}_w$). *A literal l has* warrant influence *in $\mathcal{P}$, denoted by $l \leadsto^w \mathcal{P}$, if and only if there is a literal h such that either 1.) h is warranted in $\mathcal{P}$ and for every dialectical tree $\mathcal{T}^*$ rooted in an argument for h it holds that the root of $(\mathcal{T} \setminus_t l)^*$ is "defeated" or $(\mathcal{T} \setminus_t l)^*$ is empty, or 2.) h is not warranted in $\mathcal{P}$ and there is a dialectical tree $\mathcal{T}^*$ with the root being an argument for h and it holds that the root of $(\mathcal{T} \setminus_t l)^*$ is "undefeated".*

We conclude this section by providing a formal result that follows the iterative development of the notions of influences we gave above.

Proposition 2. *Given a de.l.p. $\mathcal{P}$ it holds that if $l \leadsto^w \mathcal{P}$ then $l \leadsto^\mathcal{T} \mathcal{P}$, and if $l \leadsto^\mathcal{T} \mathcal{P}$ then $l \leadsto^\mathcal{A} \mathcal{P}$.*

4 Rationality Postulates for Contraction in DeLP

The classic contraction operation $K - \phi$ for propositional logic is an operation that satisfies two fundamental properties, namely $\phi \notin Cn(K-\phi)$ and $K-\phi \subseteq K$. Hereby, the strong consequence operator of propositional logic and the resulting set of consequences $Cn(K)$ is the measure for the success of the contraction operation and therefore the scope of the considered effects of the operation. Given that we are dealing with a logic very different from propositional logic that is based on a dialectical evaluation of arguments we also have to consider a different *scope* on the effects for an appropriate contraction operation in this setting. For a de.l.p. $\mathcal{P}$ its *logical scope* is a set of literals that are relevant in a contraction scenario, i. e. that can be derived from $\mathcal{P}$ in some way, or that has some influence on $\mathcal{P}$.

Definition 8 (Logical Scope). *For $\mathcal{P}$ we define a class of* logical scopes $S_*(\mathcal{P})$ *via* $\mathcal{S}_d(\mathcal{P}) = \{l \mid P {\mid\!\sim} l\}$, $\mathcal{S}_w(\mathcal{P}) = \{l \mid P {\mid\!\sim}_w l\}$, $\mathcal{S}_{\mathcal{I}_\mathcal{A}}(\mathcal{P}) = \{l \mid l \rightsquigarrow^{\mathcal{A}} \mathcal{P}\}$, $\mathcal{S}_{\mathcal{I}_\mathcal{T}}(\mathcal{P}) = \{l \mid l \rightsquigarrow^{\mathcal{T}} \mathcal{P}\}$, *and* $\mathcal{S}_{\mathcal{I}_w}(\mathcal{P}) = \{l \mid l \rightsquigarrow^{w} \mathcal{P}\}$.

Proposition 3. *For any d.e.lp. $\mathcal{P}$ it holds that* $\mathcal{S}_w(\mathcal{P}) \subseteq \mathcal{S}_{\mathcal{I}_w}(\mathcal{P}) \subseteq \mathcal{S}_{\mathcal{I}_\mathcal{T}}(\mathcal{P}) \subseteq \mathcal{S}_{\mathcal{I}_\mathcal{A}}(\mathcal{P}) \subseteq \mathcal{S}_d(\mathcal{P})$.

Based on the notion of scopes we propose specifications of contraction operators with different scopes by sets of postulates that resemble the rationality postulates for contraction in classic belief change theory.

Definition 9. *For a de.l.p. $\mathcal{P}$ and a literal l, let $\mathcal{P} - l = (\Pi', \Delta')$ be the result of contracting $\mathcal{P}$ by l. We define the following set of postulates for different scopes $*$ with $* \in \{d, w, \mathcal{I}_\mathcal{T}, \mathcal{I}_\mathcal{A}, \mathcal{I}_w\}$.*
(Success$_*$) $l \notin \mathcal{S}_*(\mathcal{P} - l)$
(Inclusion) $\mathcal{P} - l \subseteq \mathcal{P}$
(Vacuity$_*$) *If $l \notin \mathcal{S}_*(\mathcal{P})$, then $\mathcal{P} - l = \mathcal{P}$*
(Core-retainment$_*$) *If $k \in \mathcal{P}$ (either a fact or a rule) and $k \notin \mathcal{P} - l$, then there is a de.l.p. $\mathcal{P}'$ such that $\mathcal{P}' \subseteq \mathcal{P}$ and such that $l \notin \mathcal{S}_*(\mathcal{P}')$ but $l \in \mathcal{S}_*(\mathcal{P}' \cup \{k\})$*

These postulates represent the adaptation of applicable postulates from belief base contraction [5] to de.l.p. program contraction. The first postulate defines when the contraction is considered successful, which in our case is dependent on the scope of the contraction. The second postulate states that we are actually contracting the belief base. The third postulate requires that if the literal to be contracted is out of the scope of the operator then nothing should be changed, while the fourth postulate states that only relevant facts or rules should be erased and hence demands for minimality of change.

Definition 10. *$P - l$ is called a $*$-contraction if and only if it satisfies (Success$_*$), (Inclusion), (Vacuity$_*$) and (Core-retainment$_*$) with $* \in \{d, w, \mathcal{I}_\mathcal{T}, \mathcal{I}_\mathcal{A}, \mathcal{I}_w\}$.*

In the following we are investigating constructive approaches based on kernel sets for defining $*$-contraction operations (with $* \in \{d, w, \mathcal{I}_\mathcal{T}, \mathcal{I}_\mathcal{A}, \mathcal{I}_w\}$).

Definition 11. *Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and let α be a literal. An α-*kernel *H of $\mathcal{P}$ is a set $H = \Pi' \cup \Delta'$ with $\Pi' \subseteq \Pi$ and $\Delta' \subseteq \Delta$ such that 1.) $(\Pi', \Delta') {\mid\!\sim} \alpha$, 2.) $(\Pi', \Delta') \not{\mid\!\sim} \perp$, and H is minimal wrt. set inclusion. The set of all α-kernels of $\mathcal{P}$ is called the* kernel set *and is denoted by $\mathcal{P} \perp\!\!\!\perp \alpha$.*

Note that the notion of an α-kernel is not equivalent to the notion of an argument as an argument consists only of a set of defeasible rules while an α-kernel consists of all rules needed to derive α, in particular strict rules and facts.

As in [5] we define contractions in terms of *incision functions*. A function σ is called an *incision function* if *(1)* $\sigma(\mathcal{P} \perp\!\!\!\perp \alpha) \subseteq \bigcup \mathcal{P} \perp\!\!\!\perp \alpha$ and *(2)* $\emptyset \subset H \in \mathcal{P} \perp\!\!\!\perp \alpha$ implies $H \cap \sigma(\mathcal{P} \perp\!\!\!\perp \alpha) \neq \emptyset$. This general definition for an incision function removes at least one element in every kernel set thus inhibiting every derivation of α. Such an incision function is adequate for realizing a d-contraction but it is too strict for our more general notions of contraction. We therefore drop the second condition above for incision functions used in this work.

Definition 12. *Let $\mathcal{P}$ be a de.l.p., let α be a literal, and let $\mathcal{P}\perp\!\!\!\perp\alpha$ be the kernel set of $\mathcal{P}$ with respect to α. A function σ is a* dialectical incision function *iff* $\sigma(\mathcal{P}\perp\!\!\!\perp\alpha) \subseteq \bigcup \mathcal{P}\perp\!\!\!\perp\alpha$.

Using dialectical incision functions we can define a contraction operation in DeLP as follows.

Definition 13. *Let σ be a dialectical incision function for $\mathcal{P} = (\Pi, \Delta)$. The* dialectical kernel contraction $-_\sigma$ *for $\mathcal{P}$ is defined as* $\mathcal{P}-_\sigma\alpha = (\Pi \setminus \sigma(\mathcal{P}\perp\!\!\!\perp\alpha), \Delta \setminus \sigma(\mathcal{P}\perp\!\!\!\perp\alpha))$. *Conversely, a contraction operator $\div$ for $\mathcal{P}$ is called a* dialectical kernel contraction *if and only if there is some dialectical incision function σ for $\mathcal{P}$ such that $\mathcal{P} \div \alpha = \mathcal{P}-_\sigma\alpha$ for all literals α.*

Due to lack of space we give only the definition for *$\mathcal{I}_\mathcal{T}$-incision function*, the other incision functions are defined analogously.

Definition 14. *Let (Π, Δ) be a de.l.p., α a literal, and σ be a dialectical incision function with $\sigma((\Pi, \Delta)\perp\!\!\!\perp\alpha) = S$. Then σ is an* $\mathcal{I}_\mathcal{T}$-incision function *if 1.) $\alpha \not\rightsquigarrow^{\mathcal{T}} (\Pi \setminus S, \Delta \setminus S)$ and 2.) there is no $S' \subset S$, such that S' satisfies 1.).*

Proposition 4. *Let (Π, Δ) be a de.l.p., α a literal, and let $* \in \{d, \mathcal{I}_\mathcal{A}, \mathcal{I}_\mathcal{T}, \mathcal{I}_w, w\}$. If σ is a dialectical $*$-incision function then $-_\sigma$ is a $*$-contraction.*

5 Conclusions

In this work we started the investigation of contraction operations in defeasible logic programs. We identified different approaches to contraction depending on the notion of success of the operation. Besides the notions based on entailment and warrant we elaborated on more fine grained differences based on notions of influence. This lead to the definition of rationality postulates for each type of contraction. Furthermore, we showed that each contraction operation is constructible using kernel sets in the style of classic belief contraction.

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