Towards Conditional Logic Semantics for Abstract Dialectical Frameworks

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Abstract. We take first steps towards an integrative approach of combining conditional logic semantics with abstract dialectical frameworks. More precisely, we interpret an abstract dialectical frameworks as a conditional logic knowledge base and apply the Z-inference relation in order to obtain a new semantics for abstract dialectical frameworks. We discuss some example translations and obtain a first result pertaining to a characterisation of the different notions of consistency.

1 Introduction

It is well-known that argumentation and nonmonotonic resp. default logics are closely connected: In [7] it is shown that Reiter's default logic can be implemented by abstract argumentation frameworks, a most basic form of computational model of argumentation to which many existing approaches to formal argumentation refer. On the other hand, it is clear that argumentation allows for nonmonotonic, defeasible reasoning, and in [22] computational models of argumentation are assessed by formal properties that have been adapted from nonmonotonic logics. Furthermore, answer set programming [11] as one of the most successful nonmonotonic logics has often been used to implement argumentation [8, 5]. Nevertheless, argumentation and nonmonotonic reasoning are perceived as two different fields which do not subsume each other, and indeed, often attempts to transform reasoning systems from one side into systems of the other side have been revealing gaps that could not be closed (cf., e.g., [26, 15]. While one might argue that this is due to the seemingly richer, dialectical structure of argumentation, in the end the evaluation of arguments often boils down to comparing arguments with their attackers, and comparing degrees of belief is a basic operation in qualitative nonmonotonic reasoning. Therefore, in spite of the abundance of existing work studying connections between the two fields, the true nature of the relationship between argumentation and nonmonotonic reasoning has not been fully understood.

We aim at deepening the understanding of the relationships between argumentation and nonmonotonic logics and establishing a theoretical basis for integrative approaches by focusing on most fundamental approaches on either side: *Abstract Dialectical Frameworks* (ADFs) [4] for argumentation, and *Conditional Logics* (CL) [20, 12, 23] for nonmonotonic logics. ADFs are an approach to formal argumentation, which subsumes many other argumentative formalisms in a generic, logic-based way. On the side of nonmonotonic logics, conditionals have been shown (and often used) to implement nonmonotonic inferences and provide expressive formalisms to represent knowledge bases; some of the best nonmonotonic inference systems (e. g., System Z [12]) make use of conditionals. Furthermore, they also play a basic role for belief revision which is often considered to be a dynamic counterpart to nonmonotonic logics [10]. Both ADFs and CL can be considered as high-level formalisms implementing properly the basic nature of the respective field without being restricted too much by subtleties of specific approaches, and both are based on 3-valued logics.

In this work we take a first step towards an integrative approach by using conditional logic semantics for abstract dialectical frameworks. Syntactically, both frameworks focus on pairs of objects such as (ϕ, ψ). In conditional logic, these pairs are interpreted as conditionals with the informal meaning "if ϕ is true then, usually, ψ is true as well" and written as $(\psi|\phi)$. In abstract dialectical frameworks, these pairs are interpreted as acceptance conditions, with the constraint that $\psi = a$ is a single statement, and interpreted as "if ϕ is accepted then a is accepted as well". The resemblance of these informal interpretations is striking, but both approaches use fundamentally different semantics to formalise these interpretations. Here we ask the question of whether, and how we can interpret abstract dialectical frameworks in terms of conditional logic so that acceptance in the argumentative system is defined by a nonmonotonic inference relation based on conditionals. We take some first steps towards answering this question by translating ADFs into conditional knowledge bases and applying the Z-inference relation [12] to these knowledge bases. We exemplify this translation with several examples and compare the resulting acceptance relation to the usual ADF semantics. The main theoretical contribution of this paper is a characterisation result of the applicability of conditional logic semantics, stating that an abstract dialectical framework is consistent wrt. conditional logic semantics if and only if the conjunction of each acceptance function with its conclusion is satisfiable.

With this paper, we continue work relating argumentation and conditional reasoning that was started in [14] where DeLP rules [9] are interpreted as conditionals; hence, a DeLP program immediately corresponds to a conditional knowledge base, and nice relationships between DeLP acceptance and system Z inference could be shown. The results of that paper encouraged us to broaden these investigations to other argumentation systems. Due to their generality and their non-classical, 3-valued semantics, ADFs seem to be perfectly suited for taking first steps towards a more general framework.

The rest of this paper is organised as follows. In Section 2 we provide some necessary preliminaries. Section 3 contains our main contribution by formalising our approach to applying conditional logic semantics on abstract dialectical frameworks, discussing several examples on this application, and stating our main result. In Section 4 we discuss some related works and we conclude in Section 5 with a summary.

2 Preliminaries

In the following, we we briefly recall some general preliminaries on propositional logic (Section 2.1), as well as technical details on ADFs [4] (Section 2.2) and conditional logic (Section 2.3).

2.1 Propositional Logic

For a set At of atoms let $\mathcal{L}(\mathsf{At})$ be the corresponding propositional language constructed using the usual connectives $\wedge$ (*and*), $\vee$ (*or*), and $\neg$ (*negation*). A (classical) *interpretation* (also called *possible world*) ω for a propositional language $\mathcal{L}(\mathsf{At})$ is a function $\omega : \mathsf{At} \rightarrow \{\mathsf{T}, \mathsf{F}\}$. Let $\Omega(\mathsf{At})$ denote the set of all interpretations for At. We simply write Ω if the set of atoms is implicitly given. An interpretation ω *satisfies* (or is a *model* of) an atom $a \in \mathsf{At}$, denoted by $\omega \models a$, if and only if $\omega(a) = \mathsf{T}$. The satisfaction relation $\models$ is extended to formulas as usual.

As an abbreviation we sometimes identify an interpretation ω with its *complete conjunction*, i. e., if $a_1, \ldots, a_n \in \mathsf{At}$ are those atoms that are assigned T by ω and $a_{n+1}, \ldots, a_m \in \mathsf{At}$ are those propositions that are assigned F by ω we identify ω by $a_1 \ldots a_n \overline{a_{n+1}} \ldots \overline{a_m}$ (or any permutation of this). For example, the interpretation ω_1 on $\{a, b, c\}$ with $\omega(a) = \omega(c) = \mathsf{T}$ and $\omega(b) = \mathsf{F}$ is abbreviated by $a\bar{b}c$.

For $\Phi \subseteq \mathcal{L}(\mathsf{At})$ we also define $\omega \models \Phi$ if and only if $\omega \models \phi$ for every $\phi \in \Phi$. Define the set of models $\mathsf{Mod}(X) = \{\omega \in \Omega(\mathsf{At}) \mid \omega \models X\}$ for every formula or set of formulas X. A formula or set of formulas X_1 *entails* another formula or set of formulas X_2, denoted by $X_1 \models X_2$, if $\mathsf{Mod}(X_1) \subseteq \mathsf{Mod}(X_2)$.

Finally, let $Cn(\Phi)$ denote the deductive closure of a set $\Phi \subseteq \mathcal{L}(\mathsf{At})$, i. e., $Cn(\Phi) = \{\phi \mid \Phi \models \phi\}$.

2.2 Abstract Dialectical Frameworks

Abstract Dialectical Frameworks generalise abstract argumentation frameworks [7] and provide a general framework to discuss various issues in formal argumentation such as preferences [4] and support [25]. An ADF D is a tuple $D = (S, L, C)$ where S is a set of statements, $L \subseteq S \times S$ is a set of links, and $C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par_D(s)} \rightarrow \{\mathsf{T}, \mathsf{F}\}$ for each $s \in S$ with $par_D(s) = \{s' \in S \mid (s', s) \in L\}$ (acceptance functions). An acceptance function C_s defines the cases when the statement s can be accepted (truth value T), depending on the acceptance status of its parents in D. By abuse of notation, we will often identify an acceptance function C_s by its equivalent *acceptance condition* which models the acceptable cases as a propositional formula. In other words, we assume $C_s \in \mathcal{L}(S)$.

Example 1. Consider an ADF $D_1 = (S_1, L_1, C_1)$ with

$$\begin{aligned} S_1 &= \{a, b, c, d\} \\ L_1 &= \{(a, b), (b, a), (b, d), (c, d)\} \\ C_1 &= \{C_a, C_b, C_c, C_d\} \end{aligned}$$

with $C_a = \neg b$, $C_b = \neg a$, $C_c = \top$, and $C_d = c \wedge \neg b$. The framework D is depicted as a graph in Figure 1. Informally speaking, the acceptance conditions can be read as "a is accepted if b is not accepted", "b is accepted if a is not accepted", "c is always accepted", and "d is accepted if c is accepted and b is not accepted". Figure 1 shows a graphical depiction of the ADF D_1, where next to each node its acceptance condition is given.

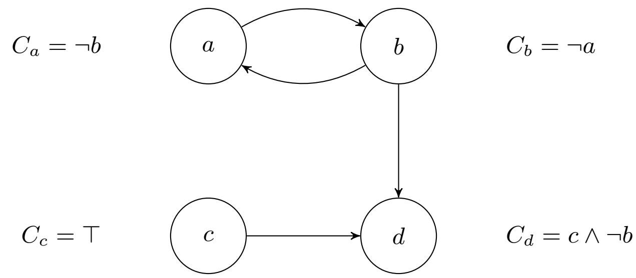

Fig. 1. A abstract dialectical framework

An ADF $D = (S, L, C)$ is interpreted through 3-valued interpretations $v : S \to \{\mathsf{T}, \mathsf{F}, \mathsf{U}\}$, which assign to each statement in S either the value T (accepted), F (rejected), or U (undecided, unknown). A 3-valued interpretation v can be extended to arbitrary propositional formulas over S via

1. $v(\neg\phi) = \mathsf{F}$ iff $v(\phi) = \mathsf{T}$, $v(\neg\phi) = \mathsf{T}$ iff $v(\phi) = \mathsf{F}$, and $v(\neg\phi) = \mathsf{U}$ iff $v(\phi) = \mathsf{U}$;
2. $v(\phi \wedge \psi) = \mathsf{T}$ iff $v(\phi) = c(\psi) = \mathsf{T}$, $v(\phi \wedge \psi) = \mathsf{F}$ iff $v(\phi) = \mathsf{F}$ or $v(\psi) = \mathsf{F}$, and $v(\phi \wedge \psi) = \mathsf{U}$ otherwise;
3. $v(\phi \vee \psi) = \mathsf{T}$ iff $v(\phi) = \mathsf{T}$ or $v(\psi) = \mathsf{T}$, $v(\phi \vee \psi) = \mathsf{F}$ iff $v(\phi) = c(\psi) = \mathsf{F}$, and $v(\phi \vee \psi) = \mathsf{U}$ otherwise.

Then v is a *model* of D if for all $s \in S$, $v(s) \neq \mathsf{U}$ implies $v(s) = v(C_s)$.

Example 2. We continue Example 1 and consider the three-valued interpretations v_1, v_2, v_3 defined via

$$
\begin{array}{llll}
v_1(a) = \mathsf{T} & v_1(b) = \mathsf{F} & v_1(c) = \mathsf{T} & v_1(d) = \mathsf{T} \\
v_2(a) = \mathsf{F} & v_2(b) = \mathsf{T} & v_2(c) = \mathsf{T} & v_2(d) = \mathsf{F} \\
v_3(a) = \mathsf{U} & v_3(b) = \mathsf{U} & v_3(c) = \mathsf{T} & v_3(d) = \mathsf{F}
\end{array}
$$

Observe that both v_1 and v_2 are models of D_1 (e. g., it holds $T = v_1(d) = v_1(C_d) = v_1(c \wedge \neg b) = \mathsf{T} \wedge \mathsf{T} = \mathsf{T}$). Observe also that v_3 is not a model as, e. g., $v_3(d) = \mathsf{F}$ but $v_3(C_d) = v_3(c \wedge \neg b) = \mathsf{T} \wedge \mathsf{U} = \mathsf{U}$.

On top of the notion of a model, various semantics can be defined for ADFs such as the grounded, complete, preferred, and stable semantics [4]. These semantics constrain the set of models further by imposing additional constraints. For that, let $\leq_i$ be the *information order* on truth values defined via $\mathsf{U} \leq_i \mathsf{T}$ and $U \leq_i \mathsf{F}$. Then $(\{\mathsf{T}, \mathsf{F}, \mathsf{U}\}, \leq_i)$ is a complete meet-semi-lattice [4] where the meet operator $\sqcap$ is defined via $\mathsf{T} \sqcap \mathsf{T} = \mathsf{T}$, $\mathsf{F} \sqcap \mathsf{F} = \mathsf{F}$, and $\alpha \sqcap \beta = \mathsf{U}$ otherwise. For two interpretations v_1, v_2 be write $v_1 \leq_i v_2$ iff $v_1(s) \leq_i v_2(s)$ for all $s \in S$ and

$$(v_1 \sqcap v_2)(s) = v_1(s) \sqcap v_2(s)$$

for all $s \in S$. For an interpretation v let $[v]_2$ be the set of interpretations v' with $v \leq_i v'$ and $U \notin \mathsf{im}\ v'$.[3] Define the operator Γ_D on interpretations via

$$\Gamma_D(v)(s) = \bigsqcap\{w(C_s) \mid w \in [v]_2\}$$

for all interpretations v and statements s. In other words, $\Gamma_D(v)$ is an interpretation, which maps every statement to the consensus of all two-valued extensions of v.

Definition 1. *Let $D = (S, L, C)$ be an ADF and v an interpretation.*

1. *v is a complete model of D iff $\Gamma_D(v) = v$.*
2. *v is the grounded model of D if it is complete and $v \leq_i v'$ for all complete models v'.*
3. *v is a preferred model of D if it is complete and there is no complete model v' with $v <_i v'$.*

Brewka and Woltran also define stable models [4], which we do not consider here.

For $\sigma \in \{gr, co, pr\}$ (grounded, complete, preferred semantics, respectively) define an inference relation $\mathrel{|\!\!\sim}^{\sigma}_{\text{cr}}$ via $D \mathrel{|\!\!\sim}^{\sigma}_{\text{cr}} a$ iff $a \in S$ is mapped to T in some σ-model of D. Considering a skeptical reasoning perspective, we can define an inference relation $\mathrel{|\!\!\sim}^{\sigma}_{\text{sk}}$ via $D \mathrel{|\!\!\sim}^{\sigma}_{\text{sk}} a$ iff $a \in S$ is mapped to T in all σ-models of D. As there is a uniquely defined grounded model, both inference relations collapse for grounded semantics and we simply write $\mathrel{|\!\!\sim}^{gr}$ instead of $\mathrel{|\!\!\sim}^{gr}_{\text{cr}}$ or $\mathrel{|\!\!\sim}^{gr}_{\text{sk}}$.

2.3 Conditional Logic

A conditional of the form $(\psi|\phi)$ connects two formulas, the antecedence ϕ and the conclusion ψ (often in a meaningful way) and represents a rule "*If ϕ then (usually, probably) ψ*". An important property of conditionals in general is their *defeasibility*, i. e., the possibility of the conclusion of a conditional being overruled when more information becomes available. Formalising defeasibility is one of the core challenges in knowledge representation and reasoning, and there are plenty of approaches that aim at addressing this, see e. g. [21, 17, 16] for some examples. We will take conditionals as basic formal entities for (nonmonotonic, plausible) reasoning. There are many different conditional logics (cf., e. g., [17, 20]), we will just use basic properties of conditionals that are common to many conditional logics and are especially important for nonmonotonic reasoning: Basically, we follow the approach of de Finetti [6] who considered conditionals as *generalized indicator functions* for possible worlds ω and define:

$$(\psi|\phi)(\omega) = \begin{cases} 1 & : \quad \omega \models \phi \wedge \psi \\ 0 & : \quad \omega \models \phi \wedge \neg\psi \\ u & : \quad \omega \models \neg\phi \end{cases} \tag{1}$$

where u stands for *unknown* or *indeterminate* (not to be confused with the truth value U from the previous section). In other words, a propositional interpretation ω *verifies* a conditional $(\psi|\phi)$ iff it satisfies both antecedence and conclusion ($(\psi|\phi)(\omega) = 1$);

[3] $\mathsf{im}\ f$ is the image of f

it *falsifies* it iff it satisfies the antecedence but not the conclusion ($(\psi|\phi)(\omega) = 0$); otherwise the conditional is *not applicable*, i. e., the interpretation does not satisfy the antecedence ($(\psi|\phi)(\omega) = u$). If $(\psi|\phi)(\omega) \neq 0$, we also say that ω *satisfies* $(\psi|\phi)$. Hence, conditionals are three-valued logical entities and thus extend the binary setting of classical logics substantially in a way that is compatible with the probabilistic interpretation of conditionals as conditional probabilities. Such a conditional $(\psi|\phi)$ can be accepted as plausible if its verification $\phi \wedge \psi$ is more plausible than its falsification $\phi \wedge \neg\psi$, where plausibility is often modelled by a total preorder on possible worlds. This is in full compliance with nonmonotonic inference relations $\phi \mathrel{|\!\sim} \psi$ [18] expressing that from ϕ, ψ may be plausibly derived. A particular convenient implementation of total preorders are *ordinal conditional functions (OCFs)*, (also called *ranking functions*) $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$ [23] on the set of possible worlds Ω. They express degrees of (im)plausibility of possible worlds and propositional formulas ϕ by setting $\kappa(\phi) := \min\{\kappa(\omega) \mid \omega \models \phi\}$. OCFs κ are a very popular formal environment for nonmonotonic and conditional reasoning, allowing for simply expressing the acceptance of conditionals and nonmonotonic inferences via stating that $(\psi|\phi)$ is accepted by κ iff $\phi \mathrel{|\!\sim}_{\kappa} \psi$ iff $\kappa(\phi \wedge \psi) < \kappa(\phi \wedge \neg\psi)$, implementing formally the intuition of conditional acceptance based on plausibility mentioned above. For an OCF κ, *Bel* (κ) denotes the propositional beliefs that are implied by all most plausible worlds, i. e., *Bel* $(\kappa) = Cn(\kappa^{-1}(0))$. A set Δ of conditionals is *consistent* if there is an OCF accepting all conditionals in Δ.

We denote with CL the framework of reasoning from conditional knowledge bases Δ based on OCFs that are so-called (ranking) models of Δ, i. e., which accept all conditionals in Δ. Specific examples of such ranking models are system Z yielding the inference relation $\mathrel{|\!\sim}^{\mathrm{Z}}$ [12] and c-representations providing the basis for c-inference relations [13, 1]. In this paper, we consider the relation $\mathrel{|\!\sim}^{\mathrm{Z}}$ defined as follows. A conditional $(\psi|\phi)$ is *tolerated* by Δ if there is a possible world ω with $(\psi|\phi)(\omega) = 1$ and $(\psi'|\phi')(\omega) \neq 0$ for all $(\psi'|\phi') \in \Delta$, i. e., ω verifies $(\psi|\phi)$ and does not falsify any (other) conditional in Δ. The Z-partitioning $(\Delta_0, \ldots, \Delta_n)$ of Δ is defined as

1. $\Delta_0 = \{\delta \in \Delta \mid \Delta \text{ tolerates } \delta\}$,
2. $\Delta_1, \ldots, \Delta_n$ is the Z-partitioning of $\Delta \setminus \Delta_0$.

For $\delta \in \Delta$ define furthermore

$$Z_\Delta(\delta) = i \qquad \Longleftrightarrow \qquad \delta \in \Delta_i \text{ and } (\Delta_0, \ldots, \Delta_n) \text{ is the } Z\text{-partitioning of } \Delta$$

Finally, the ranking function κ^z_Δ is defined via

$$\kappa^z_\Delta(\omega) = \max\{Z(\delta) \mid (\psi|\phi)(\omega) = 0, (\psi|\phi) \in \Delta\} + 1$$

with $\max \emptyset = -1$. Define then $\Delta \mathrel{|\!\sim}^{\mathrm{Z}} (\psi|\phi)$ iff $\phi \mathrel{|\!\sim}_{\kappa^z_\Delta} \psi$. For a propositional formula ϕ, we have $\Delta \mathrel{|\!\sim}^{\mathrm{Z}} \phi$ iff $\Delta \mathrel{|\!\sim}^{\mathrm{Z}} (\phi|\top)$ iff $\phi \in$ *Bel* (κ^z_Δ) iff $\kappa^z_\Delta(\neg\phi) > 0$.

3 Interpreting ADFs in Conditional Logic

Given an ADF $D = (S, L, C)$, it is straightforward to derive a conditional-logic knowledge base $\Theta(D)$ defined via

$$\Theta(D) = \{(s|C_s) \mid s \in S\}$$

from D. In other words, every acceptance function of D is interpreted as a conditional. Now we can use inference relations for conditional logic on $\Theta(D)$ and see how the inferences compare to inferences made using e. g. $\mathrel{|\!\!\sim}^{gr}$ directly on D.

As a first application, in this paper we use the Z-inference relation to define an inference relation on D via

$$D \mathrel{|\!\!\sim}^{\mathrm{Z}} a \quad \text{iff} \quad \Theta(D) \mathrel{|\!\!\sim}^{\mathrm{Z}} a. \tag{2}$$

Note that $\Theta(D) \mathrel{|\!\!\sim}^{\mathrm{Z}} a$ is equivalent to stating $a \in \mathit{Bel}\,(\kappa^z_\Delta)$. The three-valued model v_Z that can be associated with this inference relation is defined by

$$v_Z(s) = \begin{cases} \mathsf{T} \text{ if } s \in \mathit{Bel}\,(\kappa^z_\Delta), \\ \mathsf{F} \text{ if } \overline{s} \in \mathit{Bel}\,(\kappa^z_\Delta), \\ \mathsf{U} \text{ otherwise.} \end{cases} \tag{3}$$

We illustrate this new inference relation with some examples which are taken from [4].

Example 3. Let $D_2 = (S_2, L_2, C_2)$ with

$$S_2 = \{a, b, c\} \qquad L_2 = \{(a, b)\} \qquad C_2 = \{C_a = \top, C_b = \neg a \vee c, C_c = b\}$$

Then $\Delta = \Theta(D_2) = \{(a|\top), (b|\neg a \vee c), (c|b)\}$, and it is easily checked that the tolerance partitioning is just $\Delta_0 = \Delta$. This means $\kappa^z_\Delta(abc) = \kappa^z_\Delta(a\overline{b}\overline{c}) = 0$ and $\kappa^z_\Delta(\omega) = 1$ for all other worlds ω because each such ω falsifies at least one conditional from $\Theta(D)$. Therefore, $\mathit{Bel}\,(\kappa^z_\Delta) = Cn(a(bc \vee \overline{b}\overline{c}))$, In terms of arguments, this yields $D_2 \mathrel{|\!\!\sim}^{\mathrm{Z}} a$, and $v_Z(a) = \mathsf{T}, v_Z(b) = v_Z(c) = \mathsf{U}$.

Example 4. Let $D_3 = (S_3, L_3, C_3)$ with

$$S_3 = \{a, b, c\} \qquad L_3 = \{(a, c), (b, c)\} \qquad C_3 = \{C_a = c, C_b = c, C_c = a \Leftrightarrow b\}$$

We obtain $\Delta = \Theta(D_3) = \{(a|c), (b|c), (c|a \Leftrightarrow b)\}$, and again, the tolerance partitioning is just $\Delta_0 = \Delta$. This means that κ^z_Δ assigns the value 0 exactly to the worlds $abc, a\overline{b}\overline{c})$, and $\overline{a}b\overline{c}$, and 1 to all other worlds. in this case, none of a, b, c is in $\mathit{Bel}\,(\kappa^z_\Delta)$. The model v_Z assigns U to all arguments.

Finally, we apply the Z-inference relation to our Example 1.

Example 5. Consider the ADF D_1 from Example 1. The Θ-translation yields the conditional knowledge base $\Delta = \Theta(D_1) = \{(a|\overline{b}), (b|\overline{a}), (c|\top), (d|\overline{b}c)$ with $\Delta_0 = \Delta$. Therefore, exactly the worlds $abcd$, $abc\overline{d}$, $\overline{a}bcd$, $\overline{a}bc\overline{d}$, and $a\overline{b}cd$ are assigned 0 by κ^z_Δ, and we have $\mathit{Bel}\,(\kappa^z_\Delta) = Cn(bc \vee a\overline{b}cd) = Cn(c(b \vee a\overline{b}d))$. In terms of arguments, this means $D_1 \mathrel{|\!\!\sim}^{\mathrm{Z}} c$, and $v_Z(a) = v_Z(b) = v_Z(d) = \mathsf{U}$, while $v_Z(c) = \mathsf{T}$.

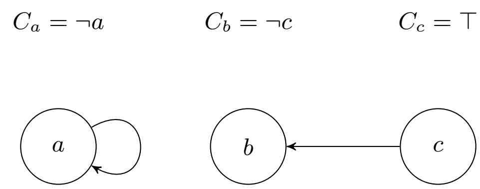

Fig. 2. The abstract dialectical framework from Example 6.

Note that in all three examples, the $\mid\!\sim^{Z}$-inferred arguments coincide with the grounded semantics. This is, in general, not true as the following example shows.

Example 6. Let $D_4 = (S_4, L_4, C_4)$ with

$$S_4 = \{a, b, c\} \quad L_4 = \{(a, a), (c, b)\} \quad C_4 = \{C_a = \neg a, C_b = \neg c, C_c = \top\}$$

which is also depicted in Figure 2. The Θ-translation yields the conditional knowledge base $\Theta(D_4) = \{(a|\overline{a}), (b|\overline{c}), (c|\top)\}$ which is inconsistent as there can be no ranking function κ that accepts the first conditional. Therefore we obtain $\Theta(D) \not\mid\!\sim^{Z} a$, $\Theta(D) \not\mid\!\sim^{Z} b$, and $\Theta(D) \not\mid\!\sim^{Z} c$. However, the grounded model of the initial ADF is able to infer c.

We say that an ADF $D = (S, L, C)$ is *sound* iff $s \wedge C_s$ is satisfiable for every $s \in S$. Then the observation from the previous example can be generalised as follows.

Proposition 1. *If ADF $D = (S, L, C)$ is not sound then $\Theta(D)$ is inconsistent.*

Proof. If D is not sound then there is an acceptance condition C_s such that $s \wedge C_s$ is unsatisfiable. For the conditional $(s|C_s) \in \Theta(D)$ this means that there is no world verifying $(s|C_s)$ and therefore $\kappa(s \wedge C_s) = \infty$ for every OCF κ. Therefore κ cannot accept $(s|C_s)$.

Example 7. Let $D_5 = (S_5, L_5, C_5)$ with

$$S_5 = \{a, b, c\} \quad L_5 = \{(b, a), (c, b), (a, c)\} \quad C_5 = \{C_a = \neg b, C_b = \neg c, C_c = \neg a\}$$

This ADF models a classical issue in argumentation: a cycle with an odd number of arguments (see Figure 3). In this case, no argument can be defended and for $\sigma \in \{gr, co, pr\}$ no argument can be inferred credulously nor skeptically. The Θ-translation yields the conditional knowledge base $\Delta = \{(a|\overline{b}), (b|\overline{c}), (c|\overline{a})\}$ with $\Delta_0 = \Delta$. Therefore, the worlds abc, $ab\overline{c}$, $a\overline{b}c$, and $\overline{a}bc$ are assigned 0 by κ^z_Δ, yielding $\Theta(D) \not\mid\!\sim^{Z} a$, $\Theta(D) \not\mid\!\sim^{Z} b$, and $\Theta(D) \not\mid\!\sim^{Z} c$ as well.

So far, all examples gave us a trivial Z-partitioning (or none at all). However, this is also not always the case as the following example shows.

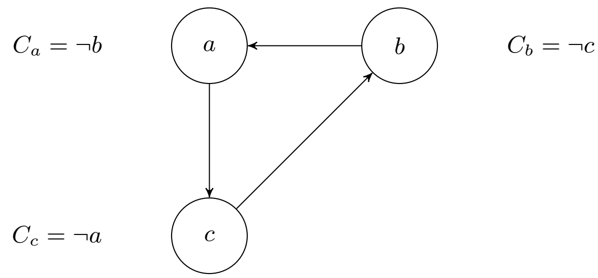

Fig. 3. The abstract dialectical framework from Example 7

Example 8. Let $D_6 = (S_6, L_6, C_6)$ with

$$S_6 = \{a, b, c\} \quad L_6 = \{(b, a), (c, b), (a, c)\} \quad C_6 = \{C_a = \neg b \wedge \neg c, C_b = \neg c, C_c = \neg a\}$$

Then $\Theta(D_6) = \{(a|\overline{b}\overline{c}), (b|\overline{c}), (c|\overline{a})\}$ and the Z-partitioning of $\Theta(D_6)$ is (Δ_0, Δ_1) with

$$\Delta_0 = \{(b|\overline{c}), (c|\overline{a})\}$$
$$\Delta_1 = \{(a|\overline{b}\overline{c})\}$$

Accordingly, we get

$$\kappa^z_\Delta(abc) = 0$$
$$\kappa^z_\Delta(ab\overline{c}) = 0$$
$$\kappa^z_\Delta(a\overline{b}c) = 0$$
$$\kappa^z_\Delta(a\overline{b}\overline{c}) = 1$$
$$\kappa^z_\Delta(\overline{a}bc) = 0$$
$$\kappa^z_\Delta(\overline{a}b\overline{c}) = 1$$
$$\kappa^z_\Delta(\overline{a}\overline{b}c) = 0$$
$$\kappa^z_\Delta(\overline{a}\overline{b}\overline{c}) = 2$$

and therefore $\Theta(D_6) \not\mathrel{|}\!\!\sim^{Z} a$, $\Theta(D) \not\mathrel{|}\!\!\sim^{Z} b$, and $\Theta(D_6) \not\mathrel{|}\!\!\sim^{Z} c$.

Let us now consider the converse of Proposition 1; we first prove a lemma that will be helpful to show that sound ADFs induce consistent conditional knowledge bases.

Lemma 1. *Let Δ be some conditional knowledge base and ψ, ϕ, ξ formulas. If $\Delta \cup \{(\psi|\phi), (\psi|\xi)\}$ is consistent then $\Delta \cup \{(\psi|\phi \vee \xi)\}$ is consistent.*

Proof. *Assume $\Delta_1 = \Delta \cup \{(\psi|\phi), (\psi|\xi)\}$ is consistent. In order to show consistency of $\Delta_2 = \Delta \cup \{(\psi|\phi \vee \xi)\}$ it suffices to show that there is at least one conditional in every $\emptyset \neq \Delta' \subseteq \Delta_2$, which is tolerated by Δ', cf. Theorem 4 in [12].*

1. *Case 1 $(\psi|\phi \vee \xi) \notin \Delta'$: Then $\Delta' \subseteq \Delta_1$ and the claim follows from the consistency of Δ_1.*

2. *Case 2 $(\psi|\phi \vee \xi) \in \Delta'$: Define $\Delta'' = \Delta' \setminus \{(\psi|\phi \vee \xi)\} \cup \{(\psi|\phi), (\psi|\xi)\} \subseteq \Delta_1$. Due to the consistency of Δ_1 there is a conditional $\delta \in \Delta''$ which is tolerated by Δ''.*
   (a) *Case 2.1 $\delta \neq (\psi|\phi), \delta \neq (\psi|\xi)$: Let ω be the world that verifies δ and satisfies all conditionals in Δ''. Then either $\omega \models \psi$ (in that case ω necessarily satisfies $(\psi|\phi \vee \xi)$ as well) or $\omega \not\models \phi$ and $\omega \not\models \xi$. In the latter case it follows $\omega \not\models \phi \vee \xi$ and therefore ω also satisfies $(\psi|\phi \vee \xi)$. It follows that δ is also tolerated by Δ'.*
   (b) *Case 2.2 $\delta = (\psi|\phi)$ (analogously for $\delta = (\psi|\xi)$). Let ω be the world that verifies δ and satisfies all conditionals in Δ''. Then $\omega \models \psi \wedge \phi$ and therefore also accepts $(\psi|\phi \vee \xi)$. It follows that $(\psi|\phi \vee \xi)$ is also tolerated by Δ'.* □

Theorem 1. *If the ADF D is sound then $\Theta(D)$ is consistent.*

Proof. *Let $D = (S, L, C)$ be sound. Let $\Theta(D) = \{\delta_1, \ldots, \delta_n\}$ with $\delta_i = (s_i|C_i)$, $i = 1, \ldots, n$, for $S = \{s_1, \ldots, s_n\}$. As D is sound, $C_i \not\models \neg s_i$ for $i = 1, \ldots, n$. We can also assume $C_i \not\models s_i$ for $i = 1, \ldots, n$, otherwise δ_i would be verified for trivial reasons. From both statements we can therefore assume that C_i does not mention s_i at all.*

In order to show consistency of $\Theta(D)$ it suffices to show that there is at least one conditional in every $\emptyset \neq \Delta \subseteq \Theta(D)$, which is tolerated by Δ, cf. Theorem 4 in [12]. Without loss of generality assume $\Delta = \{\delta_1, \ldots, \delta_m\}$ for some $1 \leq m \leq n$ (e. g. after reordering the conditionals).

Without loss of generality, we now assume that each condition C_i, $i = 1, \ldots, m$, is a conjunction of literals. This is justified due to the following:

1. *We can first safely assume that each C_i is in disjunctive normal form as the syntactic representation does not influence semantic evaluation.*
2. *Due to Lemma 1 we can split each conditional $(\phi|\psi_1 \vee \ldots \vee \psi_k)$ into $\{(\phi|\psi_1), \ldots, (\phi|\psi_k)\}$. As we will show that the conditional knowledge base consisting of the latter conditionals is consistent, it follows by Lemma 1 that the original knowledge base is consistent as well.*

Define

$$\#neg(C_i) = \{s \in S \mid C_i \models \neg s\}$$

to be the set of statements that occur negatively in each conjunction C_i. Let $j \in \{1, \ldots, m\}$ be such that $\#neg(C_j)$ is minimal wrt. set inclusion among all $\#neg(C_i)$. Define the possible world $\hat{\omega}'$ through assigning F to all atoms in $\#neg(C_j)$ and T to all remaining ones. We claim that $\hat{\omega}'$ verifies δ_j and satisfies all other conditionals in Δ.

1. *First we show that $\hat{\omega}'$ verifies δ_j: As $\hat{\omega}'$ assigns F exactly to all atoms negatively occurring in C_j we have $\hat{\omega}' \models C_j$. Furthermore, as $C_j \not\models \neg s_j$, s_j is assigned to T in $\hat{\omega}'$. Therefore $\hat{\omega}' \models s_j \wedge C_j$.*
2. *Now, we show that $\hat{\omega}'$ satisfies every $\delta_i \in \Delta$, $i = 1, \ldots, m$: If $\hat{\omega}' \not\models C_i$ then $\hat{\omega}'$ trivially satisfies δ_i. So assume $\hat{\omega}' \models C_i$. We first show $\#neg(C_i) = \#neg(C_j)$:*

(a) We have $\#neg(C_i) \subseteq \#neg(C_j)$ for the following reasons: Let $s \in \#neg(C_i)$, i. e., $C_i \models \neg s$. As $\hat{\omega}' \models C_i$ we have $\hat{\omega}' \models \neg s$. As an atom is set to F in $\hat{\omega}'$ only if $s \in \#neg(C_j)$ the claim follows.

(b) $\#neg(C_i) \not\subset \#neg(C_j)$: this is clear as $\#neg(C_i) \subset \#neg(C_j)$ would violate the minimality of $\#neg(C_j)$.

Due to $\#neg(C_j) = \#neg(C_i)$ and the fact that $C_i \not\models \neg s_i$ it follows that $\hat{\omega}' \models s_i$ (recall that all atoms are set to T except the ones in $\#neg(C_j)$). Therefore, $\hat{\omega}'$ satisfies δ_i.

As we showed that δ_j is tolerated by Δ, we have proven the general claim of the theorem. □

Taking Proposition 1 and Theorem 1 together, we obtain the following corollary.

Corollary 1. *An* ADF *D is sound iff $\Theta(D)$ is consistent.*

The result above characterises consistency in the CL framework by a simple assumption on ADFs and is a first step towards a deeper understanding of the relationships between these two approaches.

4 Related Works

Our aim in this paper is to lay foundations of integrative techniques for argumentative and conditional reasoning. There are previous works, which have similar aims or are otherwise related to this endeavour. We will discuss those in the following.

First, there is huge body of work on *structured argumentation*, i. e., approaches to argumentative reasoning that build on rule-based knowledge bases and construct arguments from chains of reasoning. Examples of such approaches are ASPIC$^+$ [19], *Assumption-based Argumentation* (ABA) [27], deductive argumentation [3], and *Defeasible Logic Programming* (DeLP) [9]. Roughly, these approaches work as follows. Starting from a knowledge base consisting of facts and rules, arguments are identified as minimal consistent derivations of their respective claims. One argument attacks another if the claim of the former somehow contradicts the contents of the latter. Building on this notion of conflict, an abstract argumentation framework can be constructed (in e. g. the case of ASPIC$^+$) and acceptable arguments are identified using some semantics for those [7]. The claims of the acceptable arguments are then regarded as justified. As one can see, structured argumentation approaches provide a *stacked* view on formal argumentation and rule-based reasoning: syntactically, structured argumentation approaches use rule-based knowledge representation components but, semantically, rely on argumentative notions.

In [15] conditional reasoning based on System Z [12] and DeLP are combined in a novel way. Roughly, the paper provides a novel semantics for DeLP by borrowing concepts from System Z that allows using *plausibility* as a criterion for comparing the strength of arguments and counterarguments. Besnard et al. [2] develop a structured argumentation approach where general conditional logic is used as the base knowledge representation formalism. Their framework is constructed in a similar fashion as the deductive argumentation approach [3] but they also provide with *conditional contrariety* a new conflict relation for arguments, based on conditional logical terms. In [29] a new semantics for abstract argumentation is presented, which is also rooted in conditional logical terms. Building on the ranking semantics System J for conditional logic [28] a ranking interpretation for extensions is provided when arguments can be instantiated by strict and defeasible rules. In [24] Strass presents a translation from an ASPIC-style defeasible logic theory to ADFs. While actually Strass embeds one argumentative formalism (the ASPIC-style theory) into another argumentative formalism (ADFs) and shows how the latter can simulate the former, the process of embedding is similar to our approach.

5 Summary

In this paper we took some first steps towards a deeper understanding of the relationship between conditional logic and abstract dialectical frameworks, thus broadening our understanding of the fields of argumentation and nonmonotonic reasoning in general. By means of examples we showed how the Z-inference relation can be applied to dialectical frameworks and we discovering striking similarities between this approach of reasoning and classical argumentation semantics (though no formal relationship has been shown yet). Our first result concerning the characterisation of consistency in conditional logic via soundness of abstract dialectical frameworks opens the way for more investigation.

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